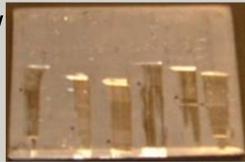


# Statistical Comparison of Tool Marks in Forensics

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## 1 Motivating Problem

- Criminal breaks into a window using a screwdriver leaving behind a tool mark.



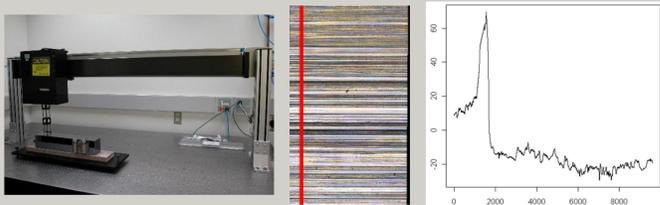
- A suspect is identified and found with a screwdriver in his possession; was that the screwdriver used in the crime?



- Grinding process of tools creates unique striae that are left behind when the tool is scraped against a hard surface, analogous to fingerprints.

## 2 Measurement Technique

Stylus profilometer reads across a tool mark recording the depths of the striae. The depths become a numerical dataset when read pixel-to-pixel.



Comparing two digitized tool marks results in a single numerical index value of similarity.

## 3 Data

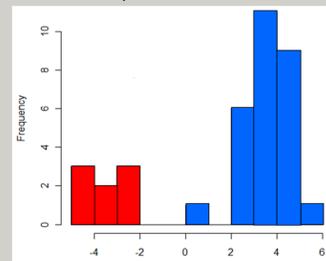
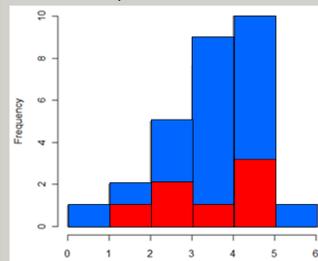
- Available tool marks
  - 1 field mark (from crime scene);  $x_0$
  - $n$  lab marks made by the suspect tool;  $x_1, \dots, x_n$
- $y_{ij}$  is the comparison value from comparing  $x_i$  with  $x_j$
- Resulting data after all pairwise comparisons are made:
  - $y_{0j}$  for  $j = 1, \dots, n$ ; all comparisons of the field mark to a lab mark
  - $y_{ij}$  for  $i, j = 1, \dots, n$  and  $i < j$ ; all pairwise comparisons of lab marks
- Regard  $y_{ij}$  as approximately normal

## 4 Interpreting the Data

Did the suspect tool create the field mark?  
i.e.: Does the field mark match the lab marks?

If  $y_{0j}$ s are comparable to  $y_{ij}$ s  $\rightarrow$  “no evidence” that the tool marks are different; i.e.: match

If  $y_{0j}$ s are small relative to  $y_{ij}$ s  $\rightarrow$  “evidence” that the tool marks are different; i.e.: no match

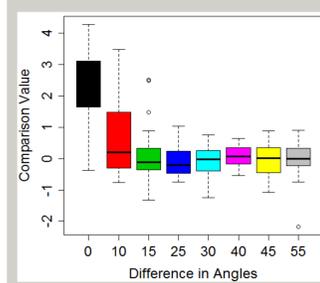


## 5 Basic Model

- $E(y_{0j}) = \mu_0$  for  $j = 1, \dots, n$ ; mean for comparison of field mark to lab marks
  - $E(y_{ij}) = \mu_1$  for  $i, j = 1, \dots, n$  and  $i < j$ ; mean for comparison of lab marks
  - $\text{Var}(y_{ij}) = \sigma^2$ ; variance for all comparisons
  - $\text{Corr}(y_{ij}, y_{kl}) = \begin{cases} 0 & \text{if } i \neq k, i \neq l, j \neq k, \text{ and } j \neq l \\ \rho & \text{if } i = k, \text{ or } i = l, \text{ or } j = k, \text{ or } j = l \\ 1 & \text{if } i = k \text{ and } j = l \end{cases}$
- That is, comparisons involving a common tool mark are correlated
- Parameters estimated using MLE based on weighted least squares
  - Likelihood ratio test used to test  $H_0: \mu_0 = \mu_1$

## 6 Angle Influence

- Angle at which the tool is held when making the mark changes the appearance of the tool mark
- Boxplots show comparison values for tool marks made by the same tool at varying angles – 30°, 45°, 60°, 75°, & 85°

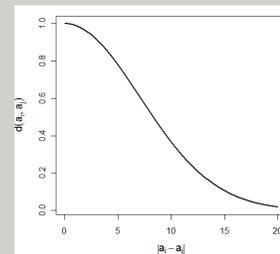


- Tool angles differing by more than 10° result in small comparison values indicative of non-matches
- Angle of a tool mark should be incorporated into the model

## 7 Model with Angle

- $a_i$  = the tool angle (in degrees) at which tool mark  $x_i$  is made

- $d(a_i, a_j) = \exp[-\theta(a_i - a_j)^2]$ , a positive similarity measure; shown in figure below



- We chose  $\theta = 0.01$  so that  $d(a_i, a_j) = 1$  when tool angles match, is small when  $|a_i - a_j| = 10$  and approaches 0 when  $|a_i - a_j| > 10$

- With more data, we would estimate  $\theta$

## 8 Analysis

Still want to know if the same tool was used to make the field mark and the lab marks. Use a LRT with the Angle Models defined as

- Null Model

$$y_{ij} \sim N(\mu_{ij}, \sigma^2) \text{ where } \mu_{ij} = \mu_0 + \alpha d(a_i, a_j) \text{ for } i, j = 0, 1, \dots, n \text{ and } i < j$$

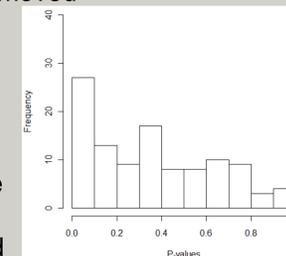
- Alternative Model

$$y_{ij} \sim N(\mu_{ij}, \sigma^2) \text{ where } \mu_{0j} = \mu_1 + \alpha_0 d(a_0, a_j) \text{ for } j = 1, \dots, n \text{ and } \mu_{ij} = \mu_1 + \alpha_1 d(a_i, a_j) \text{ for } i, j = 1, \dots, n \text{ and } i < j$$

Correlation structure is the same as described in **Block 5**. MLEs can be obtained for  $\mu_0$ ,  $\mu_1$ ,  $\sigma^2$ ,  $\alpha$ ,  $\alpha_0$  and  $\alpha_1$  provided values of  $a_0$  and  $\rho$  which are estimated using a grid search.

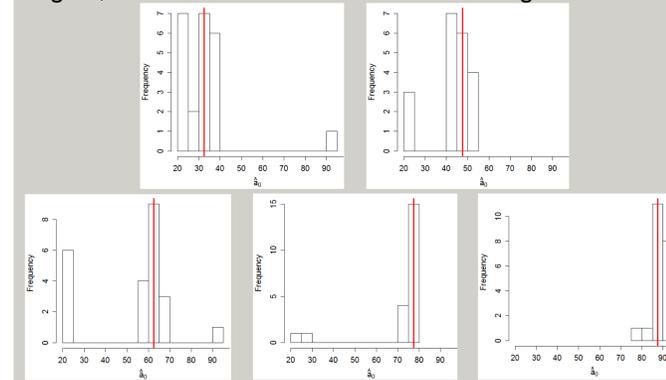
## 9 Matching Results

- Matching datasets consist of only tool marks made by the same tool; 20 tool marks – 4 at each of 5 angles
- Each mark chosen to be the field mark one-at-a-time
- Matches were found to be sensitive to flaws in the data and the matching process so “bad” tool marks and “bad” matches were removed
- LRT from **Block 8** was performed on remaining tool marks – total of 108 LRTs. Histogram shows the resulting p-values
- Since tool marks match, we expect most tests should reject  $H_0$ , so p-values should be approximately uniform which is seen in the graph



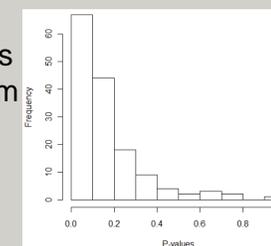
## 10 Estimation Results

The following histograms display the behavior of the estimate of  $a_0$  from the matching data. Each graph shows the estimated value of  $a_0$  for each of the five angles, the red line shows the true field angle.



## 11 Non-matching Results

- Non-matching datasets consist of 20 lab tool marks made by the same tool – 4 at each of the 5 angles – plus one field mark made by a different tool; one chosen from each angle from every remaining tool
- LRT from **Block 8** was performed on datasets
- Repeated for all 6 tools – total of 150 LRTs; p-values are shown in the histogram
- Since the tool marks do not match, we expect most of the tests should reject  $H_0$  and have small p-values which is shown in the graph



## 12 Conclusions

- The angle at which the tool is used can have a major effect on the similarity of tool marks
- For known matches, the field angle is predicted within 5° of the true angle in over 80% of the likelihood ratio tests
- Angle prediction is accurate as long as there are lab angles available within 10° of the field angle
- More data is necessary to include  $\theta$  in the estimation process
- Correct match determination is sensitive to flaws in the tool marks as well as in the initial matching process
- The LRT based on the unknown-angle model effectively discriminates between matching and non-matching tool mark pairs